## Paper / Subject Code: 86001 / Operation Research

Duration: 2 1/2 Hours Max. Marks: 75

#### Note:

- 1. All questions are compulsory. (Subject to internal Choice)
- 2. Figures to the right indicate full marks.
- 3. The normal distribution table is printed on the last page for reference.
- 4. Support your answers with diagrams/illustrations wherever necessary.
- 5. Graph paper will be supplied on request.

## Q1 A) State whether following Statements are True or False (Any Eight)

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- Linear Programming Problems consist of decision variables, an objective function, and constraints.
- In the Graphical Method of Linear Programming, the feasible region is always a polygon.
- 3. The Assignment Problem can have multiple optimal solutions.
- In a balanced transportation problem, the total supply must be equal to the total demand.
- Dummy activities are used in network diagrams to maintain logical dependencies.
- The Program Evaluation and Review Technique (PERT) considers three time estimates for activity duration.
- Crashing a project reduces both time and cost simultaneously.
- 8. In job sequencing, idle time refers to the time a machine remains unutilized.
- A two-person zero-sum game means that one player's gain is exactly equal to the other player's loss.
- In the Simplex Method, slack variables are introduced to convert inequalities into equations.

# Q1 B) Match the Column Questions: (Any SEVEN)

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| Column A                            | Column B  |
|-------------------------------------|---|
| 1. Feasible Region                  | a) Method to solve transportation problems  |
| 2. Redundant Constraint             | b) Project scheduling technique   |
| 3. Decision Variables               | c) Represents unused or excessive restrictions in LPP                               |
| 4. Unbounded Solution               | d) Graphical area satisfying all constraints in LPP                                 |
| 5. Duality in Linear<br>Programming | e) Used in Linear Programming Formulation   |
| 6. MODI Method                      | f) When there is no finite optimal solution   |
| 7. Slack Variable                   | g) Difference between primal and dual problems                                      |
| 8. Network Diagram                  | <ul> <li>h) Extra variable added to convert "≤" constraint into equality</li> </ul> |
| 9. Zero-Sum Game                    | i) One player's gain is equal to the other player's loss                            |
| 10. Least Cost Method (LCM)         | j) Heuristic approach for transportation problems                                   |

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Q2 A) Vitamins B<sub>1</sub> and B<sub>2</sub> are found in two foods F<sub>1</sub> and F<sub>2</sub>. 1 unit of F<sub>1</sub> contains 3 units of B<sub>1</sub> and 4 units of B<sub>2</sub>. 1 unit of F<sub>2</sub> contains 5 units of B<sub>1</sub> and 3 units of B<sub>2</sub>. The minimum daily prescribed consumption of B<sub>1</sub> & B<sub>2</sub> is 50 and 60 units, respectively. The cost per unit of F<sub>1</sub> & F<sub>2</sub> is Rs.6 & Rs.3, respectively.

Formulate as LPP (Linear Programming Problem).

O2 B) Solve the following Linear Programming problem by simplex method.

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Max. 
$$Z = 3x_1 + 5x_2$$

Subject to the constraints:

$$x_1 + x_3 = 4$$

$$x_2 + x_4 = 6$$

$$3x_1 + 2x_2 + x_5 = 12$$

$$X_1, X_2, X_3, X_4, X_5 \ge 0$$

Does degeneracy occur in this problem?

OR

- Q2 C) During the modification of a factory layout at BMS Auto Parts, four newly acquired machines—M<sub>1</sub>, M<sub>2</sub>, M<sub>3</sub>, and M<sub>4</sub>—need to be installed in a machine shop. The shop has five available locations: A, B, C, D, and E, which are suitable for installation. However, due to size constraints:
  - M<sub>2</sub> cannot be placed at C, and
  - M<sub>3</sub> cannot be placed at A.

The installation cost (in hundreds of rupees) for each machine at different locations is given in the following table:

| Machines       | Loc | ation |    |    |    |
|----------------|-----|-------|----|----|----|
|                | A   | В     | C  | D  | E  |
| M <sub>1</sub> | 9   | 11    | 15 | 10 | 11 |
| $M_2$          | 12  | 9     | X  | 10 | 9  |
| M <sub>3</sub> | X   | 11    | 14 | 11 | 7  |
| M4             | 14  | 8     | 12 | 7  | 8  |

Find the optimal assignment that minimizes the total installation cost.

Q2 D) Solve the following LPP by the graphical Method

Maximize  $Z=50x_1+20x_2$ 

Constraints:

 $x_1 + x_2 \le 600$ 

 $x_1 + x_2 \ge 300$ 

 $6x_1+2x_2 \ge 1200$ 

 $x_1x_2 \ge 0$ 

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Q3 A) For a project, different activities along with time and cost estimates are given below:

| Activity | Normal<br>Time<br>(Days) | Crash<br>Time<br>(Days) | Cost<br>Slope<br>(ΔC/ΔT)<br>(Rs) | Normal<br>Cost<br>(Rs) |
|----------|--------------------------|-------------------------|----------------------------------|------------------------|
| 1 - 2    | 4                        | 3                       | 30                               | 100                    |
| 1 - 3    | 6                        | 4                       | 50                               | 250                    |
| 1 - 4    | 2                        | 1                       | 20                               | 45                     |
| 2 - 4    | 5                        | 3                       | 50                               | 100                    |
| 3 - 4    | 2                        | 2                       | NIL                              | 150                    |
| 2 - 5    | 7                        | 5                       | 35                               | 120                    |
| 4 - 5    | 4                        | 2                       | 60                               | 115                    |

Indirect cost is Rs. 100 per day.

- (a) Construct the project network and identify the critical path. What is the normal duration and corresponding total cost of the project?
- (b) Systematically Crash the project and find the minimum cost and optimal time. Also, find out the additional costs required to reach the optimal time.
- (c) Find the total cost required to reach the minimum time.

A company is transporting its units from three factories F1, F2, and F3 to four warehouses W1, W2, Q3 B) W3, and W4. The transportation cost per unit (in Rs.), along with supply and demand details, is provided below. The total demand for warehouses is as follows: W1: 400 units W2: 500 units W3: 700 units W4: 800 units. The total supply available from the factories is: F1: 800 units F2: 600 units F3: 1000 units

A feasible solution, including allocations and unit cost data, is presented in the table below.

| From → To      | W <sub>1</sub> | W <sub>2</sub> | W <sub>3</sub> | W <sub>4</sub> | Supply |
|----------------|----------------|----------------|----------------|----------------|--------|
| Fı             | 12             | 6              | 20             | 25             | 800    |
|                | 300            | 500            |                |                |        |
| F2             | 6 100          | 11             | 500            | 12             | 600    |
| F <sub>3</sub> | 9              | 15             | 17             | 7              | 1000   |
| Demand         | 400            | 500            | 700            | 800            | 2400   |

- (i) Test the given solution for optimality using the Modified Distribution Method (MODI Method).
- (ii) If the solution is not optimal, modify it to obtain the best possible solution.
- (iii) Determine the minimum transportation cost.

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Q3 C) M/s Motwani Limited have taken up a special project consisting of 10 activities whose three point time estimates are listed in the table below. Activities are marked with their node numbers.

| Activity | Time Estimates in Weeks |             |             |  |  |  |  |
|----------|-------------------------|-------------|-------------|--|--|--|--|
|          | Optimistic              | Most Likely | Pessimistic |  |  |  |  |
| 1-2      | 1                       | 2           | 3           |  |  |  |  |
| 1-3      | 1                       | 2           | 3           |  |  |  |  |
| 1-4      | 1                       | 2           | 3           |  |  |  |  |
| 2-5      | 4                       | 9           | 20          |  |  |  |  |
| 3-5      | 2                       | 5           | 14          |  |  |  |  |
| 3-7      | 3                       | 6           | 15          |  |  |  |  |
| 5-7      | 1                       | 2           | 9           |  |  |  |  |
| 4-6      | 2                       | 4           | 6           |  |  |  |  |
| 6-7      | 3                       | 3           | 3           |  |  |  |  |
| 7-8      | 4                       | 4           | 4           |  |  |  |  |

- (a) Draw network diagram and find expected completion time of project.
- (b) Identity critical path.
- (c) Find the probability that the project is completed in 17 weeks.
- (d) What is the probability that the project will not be completed in 20 weeks?
- (e) If the project includes a penalty clause of Rs.1,000 per week for any delay beyond 19 weeks. What is the probability of paying a penalty of more than Rs. 5,000.
- Q4 A) Six jobs P, Q, R, S, T, and U are to be processed on two machines M and N in the order MN. The processing time (in minutes) for each job on the respective machines is given below:

| Jobs | Machine M | Machine N |
|------|-----------|-----------|
| P    | 28        | 50        |
| Q    | 20        | 35        |
| R    | 42        | 25        |
| S    | 16        | 30        |
| T    | 33        | 22        |
| U    | 26        | 45        |

#### Find:

- a) The optimal sequence of jobs to minimize total elapsed time.
- b) The total elapsed time.
- c) Idle time for each machine.

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Q4 B) Alpha Corp (Firm A) and Beta Ltd (Firm B) are two competing firms in a market. Each firm has three possible strategic choices to maximize their payoffs. The following payoff matrix represents the outcomes for Alpha Corp based on the strategic interactions with Beta Ltd.

|   |   |   | _ |  |
|---|---|---|---|--|
| ) | Į | ١ | ľ |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |
|   |   |   |   |  |

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|           | Beta I | Beta II | Beta III |
|-----------|--------|---------|----------|
| Alpha I   | 220    | 150     | 170      |
| Alpha II  | -50    | 40      | -20      |
| Alpha III | 140    | 120     | 100      |

#### Find:

- 1. Identify the optimal maximin strategy for Alpha Corp.
- 2. Determine the optimal minimax strategy for Beta Ltd.
- 3. Compute the value of the game and check if a saddle point exists.

#### OR

Q4 C) A company has 3 plants P1, P2 and P3. It supplies to 4 warehouses W1, W2, W3 and W4. O8 Cost per unit and demand – supply data is as given below. Find the Initial Feasible Solution (IFS) using the Least Cost Method (LCM).

| Plant  | W1 | W2  | W3  | W4  | Capacity |
|--------|----|-----|-----|-----|----------|
| P1     | 10 | 12  | 18  | 22  | 400      |
| P2     | 22 | 18  | 28  | 26  | 300      |
| P3     | 30 | 36  | 52  | 40  | 300      |
| Demand | 50 | 150 | 350 | 450 |          |

Q4 D) Six jobs G, H, I, J, K, L are to be processed on three machines A, B, C in the order A → B 07 → C. The processing times (in hours) are:

| Machine | G | Н | I | J  | K | L  |
|---------|---|---|---|----|---|----|
| A       | 8 | 9 | 7 | 10 | 6 | 11 |
| В       | 2 | 4 | 5 | 3  | 2 | 6  |
| C       | 5 | 7 | 4 | 6  | 3 | 8  |

#### Find:

- 1. The optimal job sequence that minimizes total elapsed time.
- 2. The total elapsed time.
- 3. Idle time on Machine A, Machine B, and Machine C.
- Q5 A) Explain Forward pass and Backward Pass calculation of Network Analysis

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Q5 B) Explain Different techniques of Operation Research.

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## Q5 C) Write Short Notes (ANY THREE)

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- 1. Vogel's Approximation Method (VAM) in Transportation Problems
- 2. North-West Corner Rule for Initial Feasible Solution
- 3. Differences Between Balanced and Unbalanced Transportation Problems
- 4. Concept of Free Float, Total Float, and Independent Float in CPM
- 5. Saddle Point and Its Significance in Game Theory

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| Normal Probability Table                         |               |         |                     |       |       |       |       |       |       |       |
|--|---------------|---------|---------------------|-------|-------|-------|-------|-------|-------|-------|
| reas under the Standard Normal Curve from 0 to z |               |         |                     |       |       |       |       |       |       |       |
|  | PHILIP I      | 22-12-1 | State of the second | 31    | 4     | 5     | 6     | 7     | 0 z   | 前衛門   |
| Ł  | 0             | 1       | 0080                | 0120  | 10160 | .0199 | .0239 | 0279  | 8     | 9     |
| 2.0  | .0000         | 0040    | 0478                | .0517 | .0557 | .0596 | .0636 | .0675 | .0319 | maco  |
| 11   | 0398          | .0438   | 0871                | .0910 | .0948 | .0987 | .1026 | 1064  | .0714 | .0753 |
| ),2  | .0793         | 0832    | 1255                | 1293  | .1331 | .1368 | 1406  | 1443  | 1103  | 1141  |
| 13.  | .1179         | 1217    | 1628                | .1664 | .1700 | .1736 | .1772 | 1808  | 1480  | .1517 |
| .5   | .1554<br>1915 | 1950    | 1985                | .2019 | .2054 | 2088  | .2123 | 2157  | 1844  | .1879 |
| .6   | .2258         | 2291    | .2324               | 2357  | .2389 | .2422 | .2454 | .2486 | 2190  | .2224 |
| .7   | .2580         | 2612    | .2642               | .2673 | .2704 | .2734 | 2764  | .2794 | .2518 | 2549  |
| 8  | .2881         | .2910   | 2939                | .2967 | .2996 | .3023 | .3051 | .3078 | .2823 | 72857 |
| 9  | .3159         | 3186    | .3212               | .3238 | .3264 | .3289 | .3315 | .3340 | .3106 | .3133 |
| 0  | .3413         | .3438   | .3461               | .3485 | .3508 | .3531 | .3554 | 3577  | .3365 | .3389 |
| 1  | .3643         | .3665   | .3686               | .3708 | .3729 | .3749 | .3770 | .3790 | .3599 | .3621 |
| 2  | 3849          | .3869   | .3888               | .3907 | .3925 | .3944 | .3962 | .3980 | .3810 | .3830 |
| 3  | .4032         | .4049   | .4066               | .4082 | .4099 | .4115 | .4131 | .4147 | .3997 | .4015 |
| 4  | .4192         | .4207   | .4222               | .4236 | .4251 | .4265 | .4279 | .4292 | .4162 | .4177 |
| 5  | .4332         | .4345   | .4357               | .4370 | .4382 | :4394 | .4406 | .4418 | -4306 | .4319 |
| 6  | .4452         | .4463   | .4474               | .4484 | .4495 | .4505 | .4515 | .4525 | .4429 | 14441 |
| 7  | .4554         | .4564   | .4573               | .4582 | .4591 | .4599 | .4608 | .4616 | .4535 | .4545 |
| 8.   | .4641         | 4649    | .4656               | 4664  | .4671 | .4678 | .4686 | .4693 | .4625 | 4633  |
| 9  | .4713         | .4719   | .4726               | .4732 | .4738 | .4744 | .4750 | .4756 | .4699 | .4706 |
| .0   | .4772         | .4778   | .4783               | .4788 | .4793 | .4798 | .4803 | .4808 | .4761 | .4767 |
| 1  | .4821         | .4826   | .4830               | .4834 | .4838 | .4842 | .4846 | .4850 | .4812 | -4817 |
| 2  | .4861         | .4864   | .4868               | .4871 | .4875 | .4878 | .4881 | .4884 | -4887 | .4857 |
| 3  | .4893         | .4896   | .4898               | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4890 |
| 4  | .4918         | .4920   | .4922               | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4916 |
| 5  | .4938         | .4940   | .4941               | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4936 |
| 6  | .4953         | .4955   | .4956               | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4952 |
| 7  | .4965         | .4966   | .4967               | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 8  | .4974         | .4975   | .4976               | .4977 | .4977 | .4978 | .4979 | .4979 | 4980  | .4981 |
| 9  | .4981         | .4982   | .4982               | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 0  | .4987         | .4987   | .4987               | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 1  | .4990         | .4991   | .4991               | .4991 | .4992 | .4992 | .4992 | .4992 | .4993 | .4993 |
| 2  | .4993         | .4993   | .4994               | .4994 | .4994 | .4994 | .4994 | .4995 | .4995 | .4995 |
| 3  | .4995         | .4995   | .4995               | .4996 | .4996 | .4996 | .4996 | .4996 | .4996 | .4997 |
| 4  | .4997         | .4997   | .4997               | .4997 | .4997 | .4997 | 4997  | .4997 | .4997 | .4998 |
| 5  | .4998         | .4998   | .4998               | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 | .4998 |
| 6  | .4998         | .4998   | .4999               | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | 4999  |
| 7  | .4999         | .4999   | .4999               | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 |
| 8  | .4999         | .4999   | .4999               | .4999 | .4999 | .4999 | .4999 | .4999 | .4999 | 4999  |
| 9  | .5000         | .5000   | .5000               | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 | .5000 |