SYIT/SEM III/Reg/Applied Mathematics

Time: 2½ Hrs. Marks:75

Note: (1) All questions are compulsory with internal choice.
(2) Figures to the right indicate full marks.
(3) Symbols have their usual meanings.
(4) Scientific calculator fx 82 series or lower version is only permitted.

Q.1 **Attempt any three of the following.** (15)

(i) Using De-Moivres theorem, find $(1+i)^8 + (1-i)^8$.

(ii) Find the general value of $Log(1+i) + Log(1-i)$.

(iii) Using De Moivre's theorem, simplify $\frac{(cos3\theta+isin3\theta)^4(cos4\theta-isin4\theta)^5}{(cos4\theta+isin4\theta)^3(cos5\theta+isin5\theta)^{-4}}$

(iv) Examine the linear dependence or independence of vectors and find the relation between them if dependent. $X_1 = [1,2,-1,0]$, $X_2 = [1,3,1,3]$, $X_3 = [4,2,1,-1]$, $X_4 = [6,1,0,-5]$.

(v) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$.

(vi) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

Q.2 **Attempt any three of the following.** (15)

(i) Solve $(6D^2 + 17D + 12)y = e^{-3x/2} + 2x$.

(ii) Solve $xyp^2 + (x^2 + y^2)p + xy = 0$.

(iii) Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(iv) Solve $\frac{d^3y}{dx^3} + y = 0$

(v) Solve $(x^2D^2 - 3xD + 5)y = x^2\sin(logx)$.

(vi) Solve $(2xy + 3x^2y^4)dx + (2x^3y^3 - x^2)dy = 0$.

Q.3 **Attempt any three of the following.** (15)

(i) Find Laplace transform of the following differential equation $y'' + 4y = sin3t$, $y(0) = y'(0) = 1$.

(ii) Find by convolution theorem $L^{-1}[\frac{1}{s^2(s+2)^2}]$.

(iii) Find $L^{-1}[\frac{s^2}{(s^2+a^2)^2}]$.

(iv) Find $L^{-1}[tan^{-1}(\frac{1}{s})]$

(v) Find $L[\cos^3 t]$

(vi) Find Laplace transform of $t^4[H(t-2)] + t^2\delta(t-2)$.

Q.4 **Attempt any three of the following.** (15)

(i) Find the area between the parabola $y = x^2 - 6x + 3$ and the line $y = 2x - 9$.

(ii) Evaluate $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dx\, dy\, dz$

(iii) Change the order of integration $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2)\, dx\, dy$

(iv) Evaluate $\iint xydxdy$ over the region bounded by the X-axis, ordinate at $x = 2a$ and the parabola $x^2 = 4ay$

(v) Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$

(vi) Change to polar co-ordinates and evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$

Q.5 **Attempt any three of the following.** (15)

(i) Evaluate $\int_0^\infty e^{-x^4}\, dx$

(ii) Prove $\lfloor\overline{n+1} = n\lfloor\overline{n}$

(iii) Prove $\text{erf}(-x) = -\text{erf}(x)$

(iv) Evaluate $\int_0^2 x^3(2-x)^{\frac{1}{2}}\, dx$

(v) State and Prove Duplication formula for Gamma function.

(vi) Prove that $\int_0^\infty \frac{cos\lambda x}{x}\left(e^{-ax} - e^{-bx}\right) dx = \frac{1}{2}\log\left(\frac{b^2+\lambda^2}{a^2+\lambda^2}\right)$, $(a > 0,\ b > 0)$

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