SYCS/SEM III/EXT/LINEAR ALGEBRA

Time: 2½ hrs. Marks:75

Note:
1. All questions are compulsory with internal choice.
2. Draw neat diagrams wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Answer the following (any four) **(20)**

(a) Evaluate : $\sqrt{16-30i}$

(b) Solve the following system of equations:

$$x+y+z=4$$
$$2x-y+3z=6$$
$$3x+2y-3z=5$$

(c) Let $S=\{(1,2,3),(1,0,1),(2,3,4)\}$. Check the linear dependancy of the set S.

(d) Express the vector $(4,6,-3)$ as a linear combination of $(1,1,1),(1,0,1)$ & $(0,1,1)$.

(e) Write a python program to rotate the complex number by angle t.

(f) Let $V=\mathbb{R}^3$ be a vector space over a field $\mathbb{R}$. Let $S=\{(x,y,z)\in\mathbb{R}^3 \mid x-y=z\}$. Show that S is vector subspace of V.

Q.2 Answer the following (any four) **(20)**

(a) Write a python program to input matrix and display inverse of that matrix.

(b) Let $A=\begin{bmatrix}3&2\\1&1\\0&5\end{bmatrix}, B=\begin{bmatrix}1&4\\2&6\\1&2\end{bmatrix}, C=\begin{bmatrix}2&2\\5&2\end{bmatrix}$ and $D=\begin{bmatrix}2&2&3&1\end{bmatrix}$

Compute the following, if they exist.

i) $A+B$ ii) $3C$ iii) $B+2D$

(c) For a linear transformation $T:U\to V$, Show that $\ker T$ is a subspace of U.

(d) State and prove Rank – Nullity theorem.

(e) Let U & V be two vector subspaces of vector space W over a field F, such that $U\cap V=\{0\}$. Show that the direct sum $U\oplus V=\{u+v \mid u\in U \,\&\, v\in V\}$ is a subspace of W.

(f) Show that $T:\mathbb{R}^3\to\mathbb{R}^2$, defined by $T(x,y,z)=(3x+y-2z, y-x)$, is a linear transformation.

Q.3 Answer the following (any four) **(20)**

(a) Write a python program to find projection of vector v on u.

(b) State and prove Pythagoras theorem in vector space.

(c) Find eigen values and eigen vectors of $\begin{pmatrix}2&1&-1\\0&1&1\\-1&1&2\end{pmatrix}$.

(d) Construct orthonormal basis of $\mathbb{R}^2$ by Gram – Schmidt process.
$v_1=(1,1)$ and $v_2=(2,5)$.

(e) Let vector $u=(1,-1,1)$ & $v=(6,3,2)$. Decompose vector $v=x+y$, such that x is parallel to u and y is orthogonal to u.

(f) Find minimal polynomial for the following matrix:

$$A=\begin{pmatrix}3&-1&0\\0&2&0\\1&-1&2\end{pmatrix}$$

Q.4 Answer the following (any five) **(15)**

(a) Express the following as $a + bi$, where $a, b \in \mathbb{R}$ & $i = \sqrt{-1}$.

$$\frac{(1+2i)(3+4i)}{(3-4i)}$$

(b) By using Cauchy's Schwartz Inequality, prove that if $\|(x, y, z)\| = 1$, then

$$\frac{3x + 6y + 2z}{7} \leq 1$$

(c) Consider the subspaces

$U = \{(x, y, z, w) : x - y = 0\}$ and

$V = \{(x, y, z, w) : x = w\}$.

Find basis and dimension of

i) U ii) V ii) $U \cap V$

(d) Let U & V be a vector spaces over a field F. Let $T: U \to V$ be a linear transformation, then Prove the following:

(i). $T(0) = 0$

(ii). For all $u, v \in U, T(u - v) = T(u) - T(v)$

(iii). For all $u, v \in U, \alpha, \beta \in F, T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$

(e) Verify Cayley Hamilton theorem for the following matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Hence find A^{-1}.

(f) Show that : $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$

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