FYIT/SEM I/CLDS

Time: 2½ Hrs. **Marks:75**

Note: **(1) All questions are compulsory with internal choice.**
(2) Figures to the right indicate full marks.
(3) Symbols have their usual meanings.
(4) Scientific calculator fx 82 series or lower version is only permitted.

Q.1 **Attempt any three of the following.** **(15)**

(i) Define partition of a set. Is $\{\{1,3,5\},\{2,6,7\},\{4,8,9\}\}$ a partition of $\{1,2,3,4,5,6,7,8,9\}$? Justify your answer.

(ii) For all sets A, B and C, Prove by using set identities: $(A-B)-(B-C)=A-B$

(iii) Indicate whether each of the following arguments is valid or invalid. Justify your answers by drawing diagrams.

(a) All human beings are mortal.
Felix is mortal.
∴ Felix is a human being.

(b) All human beings are mortal.
Zeus is not mortal.
∴ Zeus is not a human being.

(iv) Prove by contrapositive method of proof if $A \cup C \subseteq A \cap C$ then $A = C$.

(v) A study of 200 books written shows that there are three common character types: villain, female heroine and the computer genius.
128 books have a villain or a female heroine.
82 books have a female heroine.
68 books do not have a female heroine, a villain or computer genius.
54 books have a villain and a computer genius.
28 books have all three-character types.
49 books have female heroine and a computer genius.
51 books have a villain and a female heroine.

a) How many books include only a computer genius and a villain?
b) How many books does not have a computer genius?
c) How many books have a female heroine only?

(vi) Let $\mathbb{R}$ be a set of real numbers and a relation $\leq$ defined on $\mathbb{R}$. Check whether $(\mathbb{R}, \leq)$ is a POSET.

Q.2 **Attempt any three of the following.** **(15)**

(i) If $f: \mathbb{R}-\{\frac{7}{3}\} \to \mathbb{R}-\{\frac{4}{3}\}$ is defined as $f(x)=\frac{4x-5}{3x-7}$, Determine whether the function is bijective and if so, find its inverse.

(ii) Consider permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 6 & 1 & 3 & 4 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 1 & 2 & 5 \end{pmatrix}$ in S_6.
Find
a) $\tau \circ \sigma$ b) σ^{-1} c) $\sigma \circ \tau$ d) σ^2

(iii) There are three doctors , four engineers, two statistician and one economist. A committee of four from among them is to be formed. Find the probability that the committee consists of
a) one of each kind.
b) atleast one doctor.
c) economist as a member and three others.

(iv) Find the probability of getting five heads, when we toss a coin fifteen times?

(v) If X is a number appearing on the uppermost face of fair dice. Find the variance of X.

(vi) Let X be a random variable with mean 40 and standard deviation 2. Use Chebyshev's inequality to find b for which $P(40-b \leq X \leq 40+b) \geq 0.95$.

Q.3 **Attempt any three of the following.** **(15)**

(i) Find the number of distinct seven letter word that can be formed using the letters of the word "BENZENE".

(ii) Suppose five people check in their hats at a restaurant and they are given back their hats at random. Find the probabilities that no person receive his/her own hat.

(iii) Find the number of non-negative solutions $x+y+z=18$ with the condition that $x \geq 3,\ y \geq 2,\ z \geq 1$.

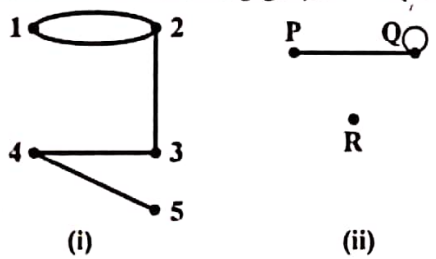
(iv) Show that however you choose five points inside an equilateral triangle of side 1, there will be atleast two points whose distance apart is less than $\frac{1}{2}$.

(v) How many positive integers, less than 100 is not a factor of 2, 3 and 5?

(vi) Solve the recurrence relation $a_n = 2a_{n-1} - 3a_{n-2}$, $a_{0=1}$, $a_1 = 2$.

Q.4 **Attempt any three of the following.** **(15)**

(i) Consider the following graphs$G = (V, E, \gamma)$ and answer the questions.



(i) (ii)

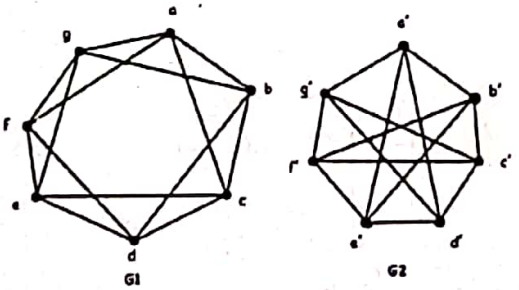
(a) Describe G formally.
(b) Find the degree of each vertex.
(c) Verify the sum of degrees and number of edges in G.

(ii) Is it possible to construct a graph with twelve nodes such that two of the nodes have degree three and the remaining nodes have degree four.

(iii) Explain the following with necessary graph.
a) Is every Eulerian graph a Hamiltonian?
b) Is every Hamiltonian graph a Eulerian?

(iv) Explain Konigsberg seven bridges problem.

(v) Determine whether the following graphs G_1 and G_2 are isomorphic.



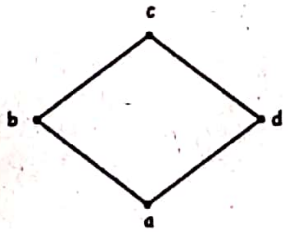
(vi) Determine the number of regions defined by a connected planar graph with 6 vertices and 10 edges. Draw a simple and a multigraph.

Q.5 **Attempt any three of the following.** **(15)**

(i) Draw Hasse diagram for the following relations on set $A = \{1,2,3,4,5\}$ with relation
$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,4),(3,5),(2,2),(3,3),(4,4),(5,5)\}$

(ii) Explain the following with an example:
a) Maximal element b) Minimal element

(iii) Determine whether the following Hasse diagram represent a lattice.



(iv) Explain BFS algorithm.

(v) Represent the expression in a binary tree and determine its value.
$$(3+9\times(15\div 5))\div\big(15-(70-(12\times 5))\big)$$

(vi) Explain post order traversal.

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