Time: 2½ hrs. **Marks:75**

Note:
1. All questions are compulsory with internal choice.
2. Draw neat diagrams wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Answer the following (any four) **(20)**

(a) Evaluate : $\sqrt{21-20i}$

(b) Solve the following system of equations:

$$2x - y - z = 5$$
$$x + 2y - 2z = 3$$
$$-x - 2y + 3z = 0$$

(c) Let $S = \{(1,2,-3), (1,-1,2), (2,3,4)\}$. Check the linear dependancy of the set S.

(d) Express the vector $(4,3,-2)$ as a linear combination of $(1,1,0), (1,0,1)\&(0,1,1)$

(e) Write a python program to rotate the complex number by angle t.

(f) Let $V = \mathbb{R}^3$ be a vector space over a field $\mathbb{R}$. Let $S = \{(x,y,z) \in \mathbb{R}^3 | x + y + z = 0\}$. Show that S is vector subspace of V.

Q.2 Answer the following (any four) **(20)**

(a) Write a python program to input matrix and display inverse of that matrix.

(b) Let $A = \begin{bmatrix} 6 & 2 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ 2 & -6 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 2 & 3 & 1 \end{bmatrix}$

Compute the following if they exist

i) $A + B$ ii) $3C$ iii) $B + 2D$

(c) For a linear transformation $f: U \rightarrow V$, Show that $\ker f$ is a subspace of U

(d) State and prove Rank – Nullity theorem.

(e) Find rank of the following matrix by using row reduce echelon form:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 & 9 \\ 2 & 5 & 8 & 11 & 14 \\ 2 & 4 & 6 & 8 & 10 \end{pmatrix}$$

(f) Show that $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$, defined by $T(x,y,z) = (x + 3y - z, x + y - z)$, is a linear transformation.

Q.3 Answer the following (any four) **(20)**

(a) Write a python program to find projection of vector v on u.

(b) State and prove Pythagoras theorem in vector space.

(c) Find eigen values and eigen vectors of $\begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.

(d) Construct orthonormal basis of $\mathbb{R}^2$ by Gram – Schmidt process.
$v_1 = (-1,2)$ and $v_2 = (3,4)$.

(e) Let vector $u = (1,-1,1)$ & $v = (4,1,2)$. Decompose vector $v = x + y$, such that x is parallel to u and y is orthogonal to u.

(f) Find minimal polynomial for the following matrix:

$$A = \begin{pmatrix} -2 & -6 & -9 \\ 3 & 7 & 9 \\ -1 & -2 & -2 \end{pmatrix}$$

Q.4 **Answer the following (any five)** **(15)**

(a) Express the following as $a + bi$, where $a, b \in \mathbb{R}$ & $i = \sqrt{-1}$

$$\frac{(2+3i)(3-4i)}{(1+2i)(1+4i)}$$

(b) Show that $F = (\mathbb{Z}_5, +, \cdot)$ is a field.

(c) Consider the subspaces

$U = \{(x, y, z, w): x + z = 0\}$ and

$V = \{(x, y, z, w): x = 0\}$.

Find basis and dimension of

1) U 2) V 3) $U \cap V$

(d) Let U & V be a vector spaces over a field F. Show that if $T: U \to V$ be a linear transformation, then $Im(T)$ is a subspace of V

(e) Verify Cayley Hamilton theorem for the following matrix:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -2 & -1 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

Hence find A^{-1}.

(f) Show that : $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$

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