FYCS/SEM II/EXT/CAL

Time: 2½ hrs. Marks:75

Note:

1. All questions are compulsory with internal choice.
2. Draw neat diagrams wherever necessary..
3. Figures to the right indicate full marks.
4. Use of scientific calculator fx 82 series and below is only allowed.

Q.1 Answer the following (any FOUR) (20)

(a) Find the interval on which $2x^3 - 9x^2 + 12x + 10$ is increasing or decreasing.

(b) Divide 80 into two parts such that their product is maximum.

(c) By using $\epsilon - \delta$ definition, prove that $\lim_{x\to 4} 3x - 5 = 7$.

(d) Using Newton's method find root of the equation $x^3 + 4x^2 - 12 = 0$ correct upto 4 decimal points. (Take $x_0 = 1$)

(e) Find $\frac{dy}{dx}$, where $xy^2 + yx^2 = \frac{1}{x} + \frac{1}{y}$.

(f) Discuss the continuity of the function in [1,3]. $f(x) = \begin{cases} 3x + 2, & x \geq 2 \\ 3x + 4, & x < 2 \end{cases}$

Q.2 Answer the following (any FOUR) (20)

(a) Evaluate by using Trapizoidal method: $\int_0^1 \frac{x^3}{x+1} dx$. (Take n = 5)

(b) Evaluate by using Simpson's method: $\int_0^1 \frac{1}{1+x^2} dx$. (Take n = 10)

(c) Solve : $\frac{dy}{dx} = \frac{x+y}{x-y}$.

(d) Solve by using Euler's method, $\frac{dy}{dx} = xy + y^2, y(1) = 1$. Find y(1.2). (Take h = 0.1)

(e) Evaluate: $I = \int e^{4x} x^2 dx$

(f) Evaluate: $I = \int_3^5 \frac{(x-3)^7}{(x-3)^7 + (5-x)^7} dx$

Q.3 Answer the following (any FOUR) (20)

(a) Using the definition of partial differentiation find f_x at (1,2) for $f(x,y) = x^2 + y^2$.

(b) Let $f(x,y,z) = x^3 + y^3 + z^3 + 3xyz$ then find f_{xy}, f_{zx} & f_{yz}.

(c) Find linearization of the following function at (1,2). $f(x,y) = x^2y + xy + xy^2$.

(d) Find directional derivative of $f(x,y) = x^2y + y^2$ at (1,1) in the direction of (3,4).

(e) Find $\frac{df}{dt}$, where $f(x,y) = x^2 + y^2$, and $x = 2t - 1, y = t^2 - 1$

(f) Find the equation of tangent & normal to the curve $x^3 + y^3 = (x+y)^2$ at (2,1)

Q.4 Answer the following (any FIVE) (15)

(a) Find equation of tangent to the curve $\frac{1}{x} + \frac{1}{y} = \frac{1}{c}$ at (2c, 2c)

(b) Find area under the curve $y = x$ and $y = x^2$.

(c) Show that $f(x) = |x|$ is not differentiable at $x = 0$.

(d) Using Newton's method find the approximate value of $\sqrt{23}$. (Take $x_0 = 5$)

(e) Using the definition of partial differentiation find f_y at (1,2) for $f(x,y) = x^3 + y$

(f) Solve the differential equation : $\frac{dy}{dx} = \frac{x^2+1}{y^2+1}$

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