FYIT/SEM II/EXT/NM

Time: 2½ hrs. **Marks:75**

Note:
1. All questions are compulsory with internal choice.
2. Figures to the right indicate full marks.
3. Symbols have their usual meanings.
4. Graph papers will be provided on request.
5. Use of scientific calculator fx 82 series and below is only allowed.

Q.1 Answer the following (Any THREE) (15)

(a) A civil engineer has measured the height of 20 storied building as 2945 cm and working height of each beam as 30 cm. While the true values are 2950 cm and 35 cm respectively. Compare the true and relative error.

(b) Evaluate $f(x) = x^3 - x^2 + x + 5$ at $x = 2.45$ using three-digit arithmetic and determine the absolute and relative errors using
(i) rounding and (ii) chopping.

(c) Explain the concept behind the conservation laws in science and engineering with examples.

(d) Use zero through third order Taylor series expansions to approximate the function $f(x) = x^3 - 3x^2 + 5x - 10$ from $x_i = 0$, $h = 1$. That is, predict the function's value at $x_{i+1} = 1$.

(e) Round off the following numbers to four significant digits.
(i) 1.6583 (ii) 30.05678 (iii) 0.70029 (iv) 3.14159 (v) 0.859378

(f) Define accuracy and precision. What are round-off errors? Explain.

Q.2 Answer the following (Any THREE) (15)

(a) Determine the real root of $26 + 85x - 91x^2 + 44x^3 - 91x^4 + x^5 = 0$ between 0.5 and 1.0 correct up to 3 decimal places using bisection method.

(b) Find the roots of the equation $2x - 3sinx - 5 = 0$ using Regula-Falsi method correct up to three decimal places.

(c) Obtain the root of the equation $xtanx = 1$ using Newton–Raphson method up to four decimal places. Take initial root as $x_0 = 4$.

(d) The following table shows the number of students and range of marks. Find the number of students who have secured less than 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	45	32	27	15

(e) Find the polynomial using Lagrange's interpolation which agrees with the tabular values given below. Hence obtain the value of $f(x)$ at $x = 2$.

x	0	1	3	4
$f(x)$	−12	0	6	12

(f) Define and express each of the Δ and ∇ in terms of E.

Q.3 Answer the following (Any THREE) (15)

(a) Solve the following simultaneous equations by Gauss-Jordan elimination method.
$2x_1 + 6x_2 - x_3 = -14, \quad 5x_1 - x_2 + 2x_3 = 29, \quad x_3 - 3x_1 - 4x_2 = 4$

(b) Solve the following simultaneous equations by Gauss–Seidel method. Perform three iterations.

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$
$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$
$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

(c) Evaluate the $\int_0^{0.3} \sqrt{1-8x^2}\,dx$ using Simpson's 3/8th rule correct up to four decimal places. Take $h = 0.05$.

(d) For the set of points $(0,1), (1,1), (2,15), (3,40), (4,85)$ evaluate $\left(\frac{dy}{dx}\right)_{x=0.5}$

(e) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$

(f) Evaluate $\int_0^2 \log(1+x)^{1/2} dx$ using Simpson's one-third rule with 8 sub-intervals.

Q.4 **Answer the following (Any THREE)** **(15)**

(a) Using Taylor series method, solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$

(b) Using modified Euler's method find the solution of $y' = \cos 2t + \sin 3t,\ 0 \le t \le 1;\ y(0) = 1$ with $h = 0.25$

(c) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, where $y(0) = 1$. Find $y(0.1)$ using Runge-Kutta second order method.

(d) Find the best-fit values of a and b so that $y = a + bx$ fits the data given in the table.

x	5	15	18	20	30	35
y	12	14	20	18	28	22

(e) Fit a second-degree parabola to the data given below.

x	2.5	3	3.5	4	4.5	5	5.5
y	4.32	4.83	5.27	5.47	6.26	6.79	7.23

(f) Obtain a regression plane equation $y = a + bx + cz$ by using multiple regression to fit the following data.

x	0	2	2.5	1	4	7
z	0	1	2	3	6	2
y	5	10	9	0	3	27

Q.5 **Answer the following (Any THREE)** **(15)**

(a) An aged person must receive 4000 units of vitamin, 50 units of minerals and 1400 calories a day. A dietician advises to thrive on foods F1 and F2 that cost Rs. 4 and Rs. 2 respectively per unit of food. If one unit of F1 contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of F2 contain 100 units of vitamins, 2 units of minerals and 40 calories. Formulate a linear programming model to minimize the cost of diet.

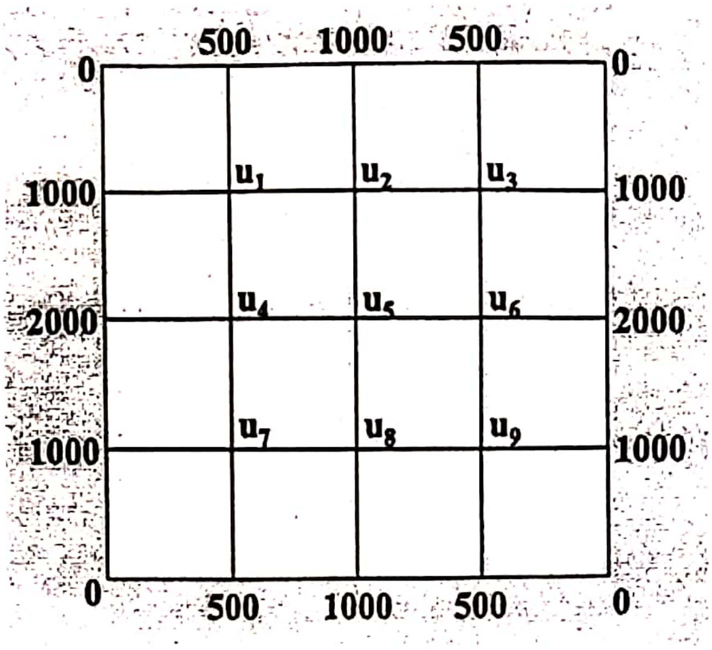
(b) A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of colour A and color B. Each gallon of color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of color B requires 1 unit of yellow dye and 6 units of green dye. Find the maximum number of gallons he can mix graphically.

(c) Classify the following partial differential equations into elliptic, parabolic and hyperbolic.

(i) $\frac{\partial^2 \phi}{\partial x^2} + (x-1)\frac{\partial^2 \phi}{\partial y^2} = 0$

(ii) $(1+x^2)\frac{\partial^2 u}{\partial x^2} + (5+2x^2)\frac{\partial^2 u}{\partial x \partial t} + (4+x^2)\frac{\partial^2 u}{\partial t^2} = 0$

(d) Given the values of $u(x, y)$ on the boundary of square mesh with boundary values as shown in the figure. Evaluate the function $u(x, y)$ satisfying the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the pivotal points of this figure by Gauss-Seidel method.



(e) Using Crank – Nicholson formula, solve $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$, given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 200t$. Compute u for one step t division. Assume $h = 0.25$.

(f) Using Bender – Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$, given $u(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = t$. Compute u for four step t division. Assume $h = 0.25$.

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