SYIT/SEM III/AM/

Time: 2½ Hrs. Marks:75

Note: (1) All questions are compulsory with internal choice.
(2) Figures to the right indicate full marks.
(3) Symbols have their usual meanings.
(4) Use of scientific calculator fx 82 series and below is only allowed.

Q.1 **Attempt any three of the following.** **(15)**

(i) Find the square root of $-5+12i$.

(ii) Find the general value of $Log(1+i)+Log(1-i)$.

(iii) Using De Moivre's theorem, simplify $\frac{(cos3\theta+isin3\theta)^4(cos4\theta-isin4\theta)^5}{(cos4\theta+isin4\theta)^3(cos5\theta+isin5\theta)^{-4}}$

(iv) Prove that $cosh^{-1}x = log\left(x+\sqrt{x^2-1}\right)$.

(v) Determine the value of λ for which the equations $3x+2y+4z=3$, $x+y+z=\lambda$, $5x+4y+6z=15$ are consistent. Find also the corresponding solution.

(vi) If $A=\begin{bmatrix}7 & 3\\ 2 & 6\end{bmatrix}$, Find A^n in terms of A.

Q.2 **Attempt any three of the following.** **(15)**

(i) Solve $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$

(ii) Solve $(x^2D^2+5xD+3)y=\left(1+\frac{1}{x}\right)^2 logx$

(iii) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=(x^2e^x)^2$

(iv) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^xsinx$

(v) Solve $(2x+e^x.logy)ydx+e^xdy=0$

(vi) Solve $\frac{d^3y}{dx^3}+y=0$

Q.3 **Attempt any three of the following.** **(15)**

(i) Find $L[\cos^3 t]$

(ii) Find $L^{-1}\left[tan^{-1}\left(\frac{1}{s}\right)\right]$

(iii) Find $L\left[\frac{e^{-2t}.sin2t.cosht}{t}\right]$

(iv) Find $L\left[\int_0^t u.e^{-3u}.cos^2 2u\,du\right]$

(v) Find $L^{-1}\left[\frac{4s+15}{16s^2-25}\right]$

(vi) Find $L^{-1}\left[\frac{1}{s.\sqrt{s+4}}\right]$

Q.4 **Attempt any three of the following.** **(15)**

(i) Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}} x^2y\,dx\,dy$

(ii) Change the order of integration $\int_0^a\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dx\,dy$

(iii) Find the volume bounded by $y^2=x$, $x^2=y$ and the planes $z=0$ and $x+y+z=1$.

(iv) Evaluate $\iiint x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x=0, y=0, z=0$, $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(v) Evaluate $\int_0^2\int_0^x\int_0^{2x+2y} e^{x+y+z}\,dx\,dy\,dz$

(vi) Find the area between the parabola $y=x^2-6x+3$ and the line $y=2x-9$.

Q.5 **Attempt any three of the following.** **(15)**

(i) Prove $\lfloor\overline{n+1}=n\lfloor\overline{n}$

(ii) Prove that $\int_0^1\sqrt{1-x^4}\,dx=\frac{\left(\lfloor\frac{1}{4}\right)^2}{6\sqrt{2\pi}}$

(iii) Prove that $\int_0^1\frac{x^a-x^b}{logx}\,dx=\log\left(\frac{a+1}{b+1}\right)$.

(iv) State and Prove Duplication formula for Gamma function.

(v) Show that $\int_0^\infty\frac{tan^{-1}\left(\frac{x}{a}\right)-tan^{-1}\left(\frac{x}{b}\right)}{x}\,dx=\frac{\pi}{2}log\frac{b}{a}$, $a>0,\ b>a$

(vi) Prove that $\int_0^\infty\frac{e^{-\beta x}.\,sin\alpha x}{x}dx=tan^{-1}\left(\frac{\alpha}{\beta}\right)$.

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