Time: 2½ hrs.

Marks:75

Note:
1. All questions are compulsory with internal choice.
2. Draw neat diagrams wherever necessary.
3. Figures to the right indicate full marks.

Q.1 **Answer the following (any four)** (20)

(a) Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = \dfrac{2x - 3}{7} \, \forall \, x \in \mathbb{R}$. Show that f is bijective. Hence find $f^{-1}(x)$

(b) Let f: $\mathbb{R} \to \mathbb{R}$, $f(x) = 3x + 1$ and g: $\mathbb{R} \to \mathbb{R}$ $g(x) = x^2 + 1$
Find   i) $g \circ f$,   ii) $f \circ f$,   iii) $g \circ g$,   iv) $f \circ g$,   v) $g \circ g \circ f$

(c) Let $\sim$ be an equivalence relation on set X. Then show that
$x \in [y] \iff [x] = [y]$, for $x, y \in X$

(d) Let R be a relation on Z, defined by xRy if and only if $5x + 6y$ is divisible by 11, for $x, y \in Z$. Show that R is an equivalence relation on Z.

(e) Solve the recurrence relation: $a_n = 3a_{n-1} + 4a_{n-2}, a_0 = 2, a_1 = 3$

(f) Find particular solution of : $a_n + 4a_{n-1} + 4a_{n-2} = 2^n$

Q.2 **Answer the following (any four)** (20)

(a) How many positive integers, less than 100 is not a factor of 2, 3 and 5?

(b) In how many ways can 5 children be arranged in a line such that
(i) Two particular children of them are always together
(ii) Two particular children of them are never together?

(c) Find the number of distinct permutations that can be formed from all the letters of the word "SOCIOLOGICAL".

(d) Find the number of non $-$ negative solutions $x + y + z = 20$ with the condition that
$x \geq 5$, $y \geq 3$, $z \geq 1$.

(e) Find middle term in the binomial expansion of $\left(2x - \dfrac{1}{x^2}\right)^4$.

(f) Define Godel number. Hence find Godel number for 12600.

Q.3 **Answer the following (any four)** (20)

(a) Consider the following graphs $G = (V, E, \gamma)$ and answer the questions.
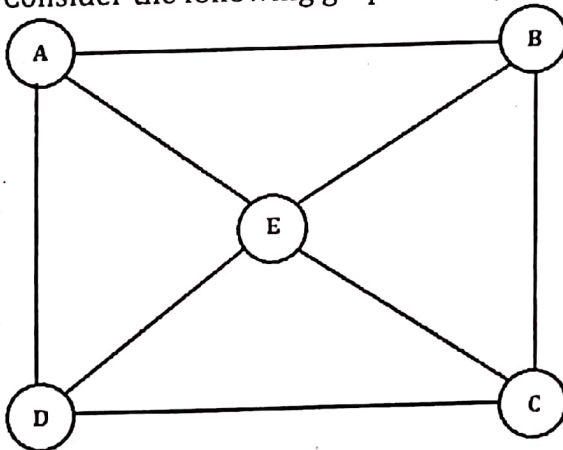
(a) Describe G formally.
(b) Find the degree of each vertex.
(c) Verify the sum of degrees and number of edges in G.

(b) Give the examples of following graph:
i) A Hamiltonian graph which is not Eulerian.
ii) An Eulerian graph which is not Hamiltonian.

(c) If 4,4,3,3,2,1,1 is a degree sequence of a graph G then find number of edges of G.

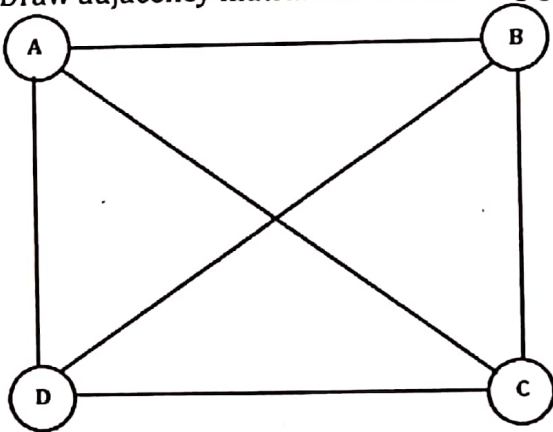(d) Show that complete graph K_n for $n \leq 4$ is planar.

(e) By using Handshaking lemma show that chemical formula of Alkane is C_nH_{2n+2} Where C is Carbon, H is Hydrogen and n is number of Carbon.

(f) Consider the inorder expression of binary search tree: $t + (u \times v) \div (w + x - y^z)$ Draw corresponding binary search tree.

(15)

Q.4 **Answer the following (any five)**

(a) Let $A = \{1,2,3\}$. Let $R: A \to A$ be a relation defined by $R = \{(1,1), (1,2), (3,2), (1,3)\}$. Find closure of R.

(b) Draw hasse diagram of relation $R = (D_{12}, | \, .)$

(c) In a class, there are 27 boys and 14 girls. The teacher wants to select 1boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection?

(d) Eight peoples are swimming in the lake. Prove that at least two of them were born on the same day of the week.

(e) Expand : $(a + b + c)^3$

(f) Draw adjacency matrix of the following graph:

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