6/10/22

SYCS/SEM III/EXT/LA

Time: 2½ hrs. Marks:75

Note:
1. All questions are compulsory with internal choice.
2. Draw neat diagrams wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Answer the following (any four) **(20)**

(a) Evaluate : $\sqrt{16-30i}$

(b) Solve the following system of equations:

$x+y+z=11$

$4x-2y+z=8$

$3x+y-2z=-3$

(c) Let $S=\{(1,2,3),(2,-1,2),(1,3,4)\}$. Check the linear dependancy of the set S.

(d) Express the vector $(2,3,4)$ as a linear combination of $(1,1,1),(1,0,1)\&(0,1,1)$

(e) Write a python program to rotate the complex number by angle t.

(f) Let $V=\mathbb{R}^3$ be a vector space over a field $\mathbb{R}$. Let $S=\{(x,y,z)\in\mathbb{R}^3 | x=y+z\}$. Show that S is vector subspace of V.

Q.2 Answer the following (any four) **(20)**

(a) Write a python program to input matrix and display inverse of that matrix.

(b) Let $A=\begin{bmatrix}2 & 2\\ -1 & 1\\ 0 & 3\end{bmatrix}, B=\begin{bmatrix}3 & 4\\ 2 & 6\\ 1 & 0\end{bmatrix}, C=\begin{bmatrix}2 & -1\\ 5 & 2\end{bmatrix}$ and $D=[2\quad 2\quad 3\quad 1]$

Compute the following if they exist

i) $A+B$ ii) $3A$ iii) $B+2D$

(c) For a linear transformation $f: U \to V$, Show that :

$\ker f = \{0\}$ if and only if f is injective.

(d) State and prove Rank – Nullity theorem.

(e) Let U & V be two vector subspaces of vector space X over a field F, such that $U\cap V=\{0\}$. Show that the direct sum $U\oplus V=\{u+v | u\in U \ \& \ v\in V\}$ is a subspace of X.

(f) Show that $T:\mathbb{R}^3\to\mathbb{R}^2$, defined by $T(x,y,z)=(2x+3y-z, y-z)$, is a linear transformation.

Q.3 Answer the following (any four) **(20)**

(a) Write a python program to find projection of vector v on u.

(b) State and prove Pythagoras theorem in vector space.

(c) Find eigen values and eigen vectors of $\begin{pmatrix}2 & 1 & -1\\ 0 & 1 & 1\\ -1 & 1 & 2\end{pmatrix}$.

(d) Construct orthonormal basis of $\mathbb{R}^2$ by Gram – Schmidt process.

$v_1=(1,2)$ and $v_2=(3,4)$.

(e) Let vector $u=(1,0,1)$ & $v=(4,1,2)$. Decompose vector $v=x+y$, such that x is parallel to u and y is orthogonal to u.

(f) Find minimal polynomial for the following matrix:

$$A=\begin{pmatrix}0 & -1 & 1\\ 1 & 2 & -1\\ 1 & 1 & 0\end{pmatrix}$$

Q.4 Answer the following (any three) **(15)**

(a) Express the following as $a + bi$, where $a, b \in \mathbb{R}$ & $i = \sqrt{-1}$

$$\frac{(2+3i)(3+4i)}{(1+2i)(3-4i)}$$

(b) Show that $F = (\mathbb{Z}_5, +, \cdot)$ is a field.

(c) Consider the subspaces

$U = \{(x, y, z, w) : x - y = 0\}$ and

$V = \{(x, y, z, w) : x = w, y = z\}$.

Find basis and dimension of

i) U ii) V ii) $U \cap V$

(d) Let U & V be a vector spaces over a field F. Show that if $T: U \to V$ be a linear transformation, then Im(T) is a subspace of V

(e) Verify Cayley Hamilton theorem for the following matrix:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -2 & -1 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

Hence find A^{-1}.

(f) Show that : $\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$

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